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Edge and bulk merons in double quantum dots with spontaneous interlayer phase coherence

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We have investigated nucleation of merons in double quantum dots when a lateral distortion with a reflection symmetry is present in the confinement potential. We find that merons can nucleate both inside and at the edge of the dots. In addition to these merons, our results show that electron density modulations can be also present inside the dots. An edge meron appears to have approximately a half integer winding number.

Keywords: spin textures; merons; density depletions.

1. Model Hamiltonian

There is a range of values of the strength of the confinement potential for which spontaneous interlayer phase coherence is stable in double quantum dots[1]. Outside this stability range the shape of the groundstate is expected to undergo a reconstruction[2]. It is the competition between the confinement potential and the Coulomb interaction which drives such a reconstruction. When the confinement potential weakens the repulsive Coulomb interaction pushes electrons from each other, and can create density depletions inside the droplet.

The interplay between the reconstruction and the formation of merons in the presence of an external potential is unclear. We investigated within a Hartree-Fock (HF) theory how merons nucleate in the groundstate when a lateral distortion with a reflection symmetry is present in the confinement potential[2]. We found that when a sufficiently large distortion is present merons can be nucleated. Both density depletions and merons can coexist in the reconstructed states. In the present work we demonstrate by integrating the electron density that these merons have total charge $\pm 1/2$. In addition we present numerical evidence for the presence of edge merons, which appear to have a half-integer winding number.

An electron in each dot is in a parabolic potential $\frac{1}{2}m^*\Omega^2 r^2$. In the strong magnetic field limit the energy of an electron with the angular momentum m in a quantum dot is $\epsilon_m = \frac{\hbar\omega_c}{2} + \hbar\omega_p(m+1)$, where ω_c is the cyclotron frequency, $\hbar\omega_p = m^*\Omega^2\ell^2$, and ℓ is the magnetic length. Hereafter we will measure $\hbar\omega_p$ in units

of $e^2/\varepsilon\ell$, using $\gamma = \hbar\omega_p/(e^2/\varepsilon\ell)$ which is a dimensionless quantity proportional to the confinement potential strength (For GaAs $\gamma = 0.131(\hbar\omega_p[meV])^2/(B[T])^{3/2}$). The real spins of all electrons are assumed to be frozen aligning with the magnetic field. We consider a distortion in the confinement potential of the dot that may be generated by attaching a metallic stripe vertically on the cylindrical metallic gate surrounding the vertical quantum dots. The vertical distance between two dots is d. Let us make a simple model for this type of distortions. We take a separable form for the distortion potential $U_{\rm dis}(\vec{r}, z) = vg(\vec{r})h(z)$, where v is the strength of the distortion. The matrix elements of this potential are

$$\langle n_1\tau_1|U_{\rm dis}|n_2\tau_2\rangle = (u_s\delta_{\tau_1\tau_2} + u_d(1-\delta_{\tau_1\tau_2}))< n_1|g(\vec{r})|n_2>, \tag{1}$$

where $|n_j\tau_j\rangle = c^\dagger_{n_j\tau_j}|0\rangle$ denotes an electron with the angular momentum n_j in the dot labeled by τ_j. The parameters u_s and u_d are defined as

$$u_s = v\int {\rm d}z h(z)\left|\varphi\left(z-\frac{d}{2}\right)\right|^2, \tag{2}$$

$$u_d = v\int {\rm d}z h(z)\varphi^*\left(z-\frac{d}{2}\right)\varphi\left(z+\frac{d}{2}\right). \tag{3}$$

The parameter u_s (u_d) denotes the strength of the matrix elements of the disorder potential between intra (inter) layer electrons. The wavefunctions $\varphi\left(z+\frac{d}{2}\right)$ and $\varphi\left(z-\frac{d}{2}\right)$ are, respectively, the left and right dot wavefunctions along the vertical direction.

The total Hamiltonian is $H = H_0 + V$, where

$$H_0 = \sum_{m\sigma}\epsilon_m c^\dagger_{m\sigma}c_{m\sigma} + \sum_{m\sigma}\sum_{m'\sigma'}\langle m\sigma|U_{\rm dis}|m'\sigma'\rangle\, c^\dagger_{m\sigma}c_{m'\sigma'}, \tag{4}$$

and

$$V = \frac{1}{2}\sum_{m_1m_2m_3m_4}\sum_{\sigma=\uparrow\downarrow}\left(\langle m_1m_2|V_{\rm s}|m_3m_4\rangle\, c^\dagger_{m_1\sigma}c^\dagger_{m_2\sigma}c_{m_3\sigma}c_{m_4\sigma}\right.$$
$$\left. + \langle m_1m_2|V_{\rm d}|m_3m_4\rangle\, c^\dagger_{m_1\sigma}c^\dagger_{m_2,-\sigma}c_{m_3,-\sigma}c_{m_4\sigma}\right). \tag{5}$$

The first term of H_0 represents the confining energy due to the parabolic potential. In the second term, $U_{\rm dis}$ is the distortion potential. In the interaction Hamiltonian V, the term V_s (V_d) represents the Coulomb interaction potential between the electrons in the same (different) dots. In the spatial coordinates, those interactions are given by $V_s(\vec{r}) = e^2/\varepsilon r$ and $V_d(\vec{r}) = e^2/\varepsilon|\vec{r} + d\hat{z}|$, respectively, where ε is the dielectric constant. The spontaneous interlayer phase coherent state (maximum density droplet state[3,4]), and meron excitations are well approximated by a HF theory[5]. The pseudospin density is computed with the calculated density matrix.

2. Results

Fig. 1 displays total electron densities for two different strengths of the lateral distortion with the center at the coordinate $(-5, 5)\ell$ by a Gaussian lateral potential

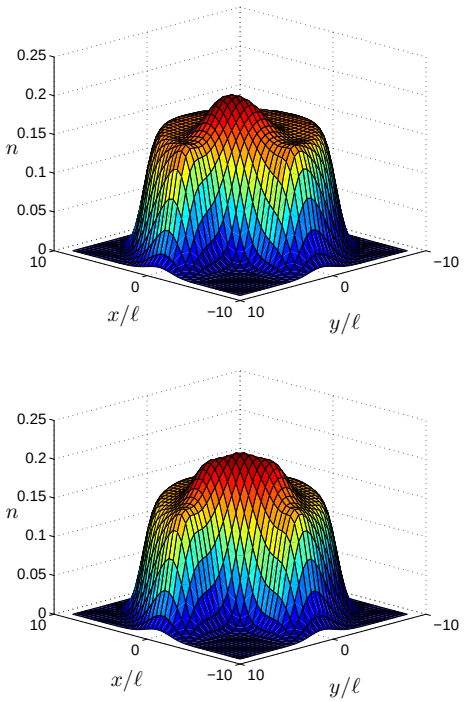

Fig. 1. Electron densities in units of $1/\ell^2$ for $(u_s, u_d) = (1.5, 0.25)(e^2/\varepsilon\ell)$ and $(u_s, u_d) = (1.8, 0.3)(e^2/\varepsilon\ell)$.

with the range $a = 3\ell$. Results for $\gamma = 0.085$ are shown for two different strengths of the distortion $(u_s, u_d) = (1.5, 0.25), (1.8, 0.3)(e^2/\varepsilon\ell)$. The maximum density is bigger than $1/2\pi\ell^2$, which implies that the total local filling factor is bigger than one where the merons are located. This is consistent with the presence of merons with negative charges on the background of the total electron density $1/2\pi\ell^2$. At $(u_s, u_d) = (1.5, 0.25)(e^2/\varepsilon\ell)$ a meron is formed with the charge $-e/2$ and vorticity $-$. For the bigger value $(u_s, u_d) = (1.8, 0.3)(e^2/\varepsilon\ell)$ two merons appear and the vorticities for the lower and upper merons are $-$ and $+$, respectively. Integrating the electron density over the range that satisfy $n(r) > \frac{1}{2\pi\ell^2}$, we find the excess charges to be $-0.54e$ and $-0.98e$, which indicates that one meron exists in the upper figure of Fig. 1 and a meron pair exists in the lower figure of Fig. 1. We have attempted to evaluate the total excess charge carried by the merons using the expression[5]

$$\Delta N = -\frac{1}{8\pi}\int \mathrm{d}^2\mathbf{r}\ \epsilon_{\mu\nu}\mathbf{m}(\mathbf{r})\cdot[\partial_\mu\mathbf{m}(\mathbf{r})\times\partial_\nu\mathbf{m}(\mathbf{r})], \tag{6}$$

where the polarization vector $\vec{m}(r)$ is $\vec{\tau}(r)/n(r)$ ($\vec{\tau}(r) = 2\vec{s}/\hbar$ and $\vec{s}$ is the pseudospin). However, this method gave inaccurate results. We believe this is because

the expression is valid only for slowly varying pseudospin fields.

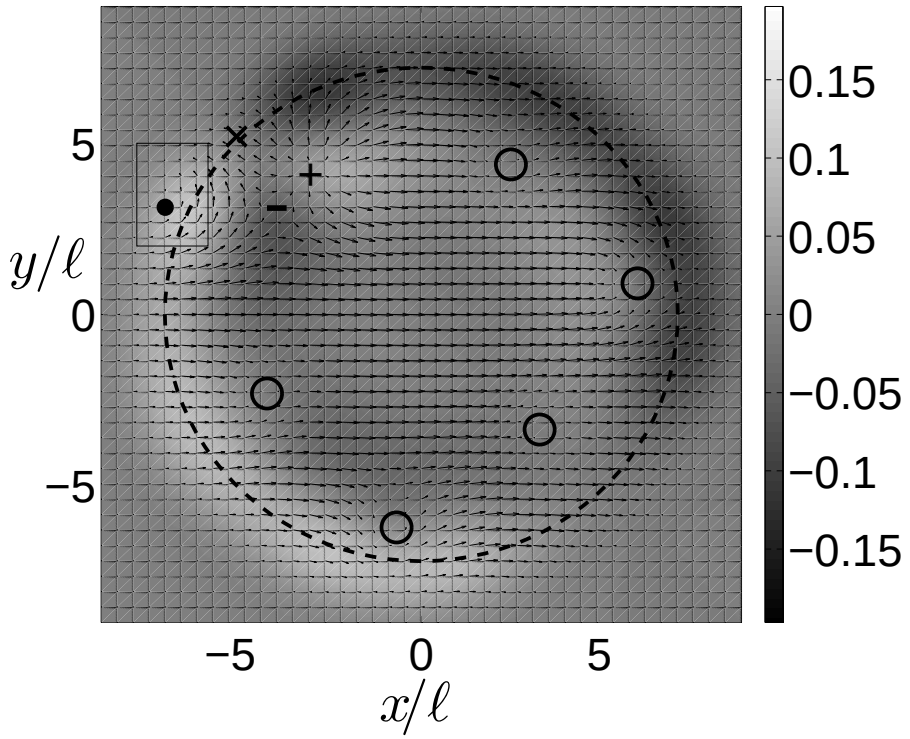

Fig. 2. Arrows are projections of $\vec{\tau}$ on the xy plane and the gray scale indicates the z-component of $\vec{\tau}$. The dashed circle is the edge of the electron droplet and + and − indicate the core positions of merons with positive and negative winding numbers. The cross sign is where the impurity is located. The black dot represents the center of an edge meron, shown in the inset, and the small circles show electron density depletions.

Fig. 2 displays how the pseudospin texture for $\gamma = 0.066$ and $(u_s, u_d) = (0.24, 0.04)(e^2/\varepsilon\ell)$. We observe merons inside the dots. The values of the polarization $m_z(r) = \tau_z(r)/n(r)$ at the meron cores are, within numerical accuracy, $+1$ and -1. This implies that the charges should be $-e/2$ since the vorticities of these two objects are opposite. The total charge of the merons is thus $-e$. Note that there are also five density depletions inside the dots (the open circles in the figure). These modulations of electron densities are not simply related to the shape of $U_{\rm para}(r) + U_{\rm dis}(r)$. Since the densities are inhomogeneous the HF single particle potential must be included into consideration. It is non-trivial to anticipate the shape of the HF single particle potential since it reflects the delicate interplay between

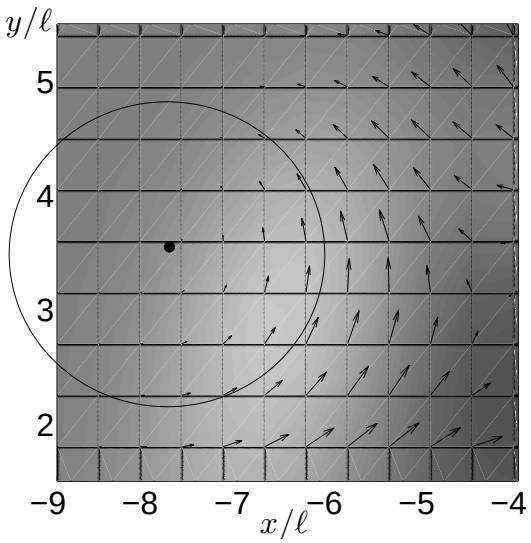

Fig. 3. Magnified view of the spin texture of an edge meron with nearly a half-integer winding number.

the Hartree self energy, exchange self energy, and confinement potential[6].

We also observe an edge meron near the edge in Fig. 2. A magnified view of the pseudospin texture is shown in Fig. 3. We draw a circle around the center of this object and see how the in-plane pseudospins rotate along the circle. The in-plane pseudospins nearly flips as the half circle is followed inside the dot. The magnitude of in-plane pseudospin values decreases to zero as the edge region is passed. We believe thus that the edge meron has approximately a half integer winding number. It is complicated to calculate accurately the excess charge of the edge meron since the electron density varies strongly near the edge even in the absence of an edge meron. It would be worthwhile to develop an analytical theory for edge merons.

References

1. Jun Hu, E. Dagotto, and A.H. MacDonald, Phys. Rev. B **54** 8616 (1996).
2. S.-R. Eric Yang and N.Y. Hwang, Phys. Rev.B **72** 165337 (2006).
3. S.-R. Eric Yang, M.D. Johnson, and A.H. MacDonald, Phys. Rev. Lett **71** 3194 (1993).
4. A.H. MacDonald, S.-R. Eric Yang, and M.D. Johnson, Aust. J. Phys. **46** 345 (1993).
5. K. Moon, H. Mori, Kun Yang, S.M. Girvin, A.H. Macdonald, L. Zheng, D. Yoshioka, and Shou-Cheng Zhang, Phys. Rev. B **51** 5138 (1995).
6. S.-R. Eric Yang and A.H. MacDonald, Phys. Rev. B **67** 155202 (2003).